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Optical domain walls

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Dynamical properties of the domain walls (DW's) in the light beams propagating in nonlinear optical fibers are considered. In the bimodal fiber, the DW, as it was recently demonstrated numerically, separates two domains with different circular polarizations. This DW is found here in an approximate analytical form. Next, it is demonstrated that the fiber's twist gives rise to an effective force driving the DW. The corresponding equation of motion is derived by means of the momentum-balance analysis, which is a technically nontrivial problem in this context (in particular, an effective mass of the DW proves to be negative). Since the sign of the twist-induced driving force depends on the DW's polarity, the DW's with opposite polarities can collide, which leads to the formation of their stable bound state. This is a domain of a certain circular polarization squeezed between semi-infinite domains of another polarization. In the absence of the twist, the DW can be driven by the Raman effect, but in this case the sign of the force does not depend on the DW's polarity and the bound state is not possible. Finally, a similar problem is considered for the dual-core fiber (coupler). In this case, the DW is a dark soliton in one core in the presence of the homogeneous field in the mate core. The dark soliton is driven by a force induced by the coupling with the mate core. The bound state of two dark solitons also exists in this system. The effects considered may find applications, e.g., for the optical storage of information.

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I. INTRODUCTION

The domain walls (DW's) furnish the simplest type of a stable defect in nonlinear patterns. Alongside the classical static DW's in ferromagnets and aniferromagnets, the study of the DW-like defects in dynamical nonlinear patterns has recently attracted a great deal of attention [1-7]. The dynamical DW's which seem most similar to the classical static ones in the magnetic systems are the walls between rolls with different orientations in the Rayleigh-Bénard convection [1, 3], or between rolls and hexagons [2, 3], or, at last, between the hexagons and the trivial state below onset [3]. These DW's may be regarded as linear defects in two-dimensional nonlinear systems, although they are actually given by solutions of effectively one-dimensional coupled Ginzburg-Landau (GL) equations [2,3]. Similar solutions describe the DW's which are kinks (phase jumps) in purely one-dimensional systems, e.g., in the GL equation with the parametric pumping [5]. Finally, quiescent and moving kinks which separate domains occupied by different stable phases (described, e.g., by the quintic GL equation [4]) can also be regarded as one-dimensional DW's.

Recently, study of the DW's was started in systems of coupled nonlinear Schrödinger (NLS) equations governing propagation of light in nonlinear optical fibers [6] and nonlinear planar lightguides [7]. In these nonlinear optical systems, the walls separate domains with different circular polarization of light. While in the analysis developed in Ref. [6] the dispersion was neglected, it was taken into account in Ref. [7]. In the latter work, two types of solutions of the coupled NLS equations were considered: periodic arrays of the DW's and solitary walls. Although the full dynamics described by coupled GL equations and by coupled NLS equations are very different (dissipative in the former case and conservative in the latter case), their static solutions coincide. The same pertains to the stability properties of those solutions. In particular, the solitary DW considered in the context of the nonlinear lightguides in Ref. [7] exactly coincides with a particular case of the solution found in Ref. [3] for the domain boundary between rolls with different orientations in the convection patterns (periodic solutions, however, were not considered in Ref. [3]). As another example of a DW in nonlinear optical fibers, it is relevant to mention the exact solution describing a full transformation of a pump wave into the Stokes wave, obtained in the framework of a system of NLS equations for the two waves coupled by the Raman interaction [20].

The main objective of this work is to consider some important properties of the polarization DW's in nonlinear optical fibers. The analysis will be based on the coupled NLS equations for the linear polarizations (while in Ref. [7] the basic polarizations were circular). Two additional important physical factors will be taken into account, viz., the linear coupling which accounts for the *twist* of the fiber and the Raman effect. In Sec. II it will be demonstrated that, in the presence of the twist, which will be treated as a small perturbation, usual static DW solutions are not possible; instead, a stable bound state of two DW's with opposite *polarities* appears. It represents a finite domain with a certain circular polarization sandwiched between semi-infinite domains of the opposite circular polarization, which may be regarded as a prediction of a dynamical state in the nonlinear optical

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fiber. The size of the intermediate domain is determined by the twist rate, as well as by the background light power. It will also be demonstrated that a solitary DW, which cannot exist in a static state, moves like a mechanical particle subjected to the action of a constant driving force proportional to the twist. The law of motion for the twist-driven DW will be derived by means of the known technique based on the balance equation for the wave momentum. However, application of this technique to the problem considered is not straightforward, so that some nontrivial tricks will be used; in particular, an effective mass of the DW proves to be negative. The prediction of the twist-induced accelerated motion of the DW formally resembles the effect well known in the single-mode nonlinear optical fibers: the constant "acceleration" of a soliton under the action of the intrapulse Raman scattering [8]. In this work, influence of the Raman effect in the *bimodal* fiber [9] will be briefly considered in Sec. III and it will be shown that it also gives rise to an acceleration of the DW (actually, this effect is physically relevant only when the temporal width of the DW in the fiber is very small, belonging to the subpicosecond region). An important difference from the twist-induced acceleration is that the sign of the "force" induced by the twist depends upon the polarity of the DW, while the Raman effect induces the force with a fixed sign. In particular, the twist-driven DW's with the opposite polarities can collide, and their collision must give rise to formation of the above-mentioned sandwich. These properties of the DW's in the twisted fiber may find applications in design of nonlinear-optical logic elements; for instance, the sandwich structure could be used for information storage.

Similar effects may take place in the directional coupler, i.e., a system of parallel tunnel-coupled monomode optical fibers (see the review paper [10]), which is considered in Sec. IV. In this system the coupling is purely linear, and it plays a role which is formally similar to that of the twist in the bimodal fiber. A structure of the DW type in the coupler can be realized as a dark soliton in one arm on the background of homogeneous cw field in the other. This structure cannot exist as a static solution, and, treating the coupling as a small perturbation, in section IV it will be shown by means of the momentum-balance technique that the coupling with the mate arm induces an effective force driving the dark soliton, the sign of the force being determined by the dark soliton's polarity. The effective mass of the dark soliton again proves to be negative. A pair of the dark solitons may form a stable bound state similar to that described above for the twisted bimodal fiber.

II. POLARIZATION DOMAINS IN THE TWISTED OPTICAL FIBER

The interaction of two orthogonal linear polarizations in the twisted nonlinear optical fiber is described by the coupled NLS equations for the envelope functions $u(z,\tau)$ and $v(z,\tau)$ [11],

$$iu_z + \frac{1}{2}u_{\tau\tau} - \left(|u|^2 + \sigma|v|^2\right)u + \kappa v = 0\,, \qquad (1)$$

$$iv_z + \frac{1}{2}v_{\tau\tau} - \left(|v|^2 + \sigma|u|^2\right)v + \kappa u = 0\,, \qquad (2)$$

where κ is proportional to the fiber's twist and the coupling constant σ is actually $\frac{2}{3}$ (the rapidly oscillating terms are, as usually, neglected). Note that the sign in front of the nonlinear terms in Eqs. (1) and (2) is chosen so that the homogeneous field will be modulationally stable [the modulationally stable NLS equations are sometimes written with the opposite signs in front of the dispersive and nonlinear terms as compared to Eqs. (1) and (2); however, it is evident that both forms of the equations are equivalent]. It is well known [16] that the nonlinear cross-phase modulation does not break the modulational stability provided that $\sigma < 1$, which is the case to be considered in this work.

In order to strongly simplify the analysis, in what follows below heavy use will be made of the evident fact that $\frac{2}{3}$ is close to the value $\sigma = 1$, at which the system of Eqs. (1) and (2) with $\kappa = 0$ is the exactly integrable Manakov's system [12]. Moreover, it is known that the system with $\sigma = 1$ and $\kappa \neq 0$ can be reduced to the Manakov's form [13]. Therefore, it will be set

$$\sigma \equiv 1 - \epsilon, \qquad (3)$$

and ϵ, actually equal $\frac{1}{3}$, will be treated as a formally small parameter. It is known that using the small ϵ strongly simplifies consideration of dynamical [14] and static [3] problems within the framework of the system of Eqs. (1) and (2). Note that an effective birefringence of the bimodal fiber in the absence of the twist is neglected in Eqs. (1) and (2). An effect of the birefringence will be briefly discussed at the end of this section.

Static solutions to Eqs. (1) and (2) are sought for in the obvious form

$$\begin{aligned} u(z,\tau) &= e^{i\beta_1 z + i\phi_0}\, U(\tau), \\ v(z,\tau) &= e^{i\beta_2 z - i\phi_0}\, V(\tau), \end{aligned} \qquad (4)$$

where β_1 and β_2 are the propagation constants of the two polarizations and ϕ_0 is an arbitrary phase constant. Actually, derivation of Eqs. (1) and (2) from Maxwell's equations assumes that the difference $\beta_1 - \beta_2$ must be small [17]. In this work, attention will be focused on the case of the equal propagation constants; the case of slightly different β_1 and β_2 will be considered briefly. Next, insertion of Eqs. (4) into Eqs. (1) and (2) leads to a system of two ordinary differential equations (ODE's) for $U(\tau)$ and $V(\tau)$ which, apart from the terms proportional to κ, has exactly the same form as in Ref. [3], where it was derived for description of the DW's in the Rayleigh-Bénard convection. As it was demonstrated in that work, it is sufficient to consider the case when the amplitudes U and V are real, and then the system of the ODE's takes the form of equations of motion for a mechanical particle with two degrees of freedom U and V and with the Hamiltonian

$$H = \frac{1}{2}\left[\left(\frac{dU}{d\tau}\right)^2 + \left(\frac{dV}{d\tau}\right)^2\right] - \frac{1}{2}\left(\beta_1 U^2 + \beta_2 V^2\right) - \frac{1}{4}\left(U^2 + V^2\right) + \frac{1}{2}\epsilon U^2 V^2 + \kappa U V\,, \quad (5)$$

where ϵ is the formal small parameter defined by Eq. (3). At $\epsilon = 0$ and $\kappa = 0$, the Hamiltonian (5) is rotationally symmetric on the plane (U, V) (i.e., it conserves the angular momentum). Therefore, to treat the terms breaking the rotational symmetry as small perturbations, it is natural to introduce the polar coordinates ρ and χ as follows [3]:

$$U \equiv \rho\cos\chi,\ V \equiv \rho\sin\chi\,. \quad (6)$$

In the zeroth approximation, i.e., at $\epsilon = 0$, and also with $\beta_1 = \beta_2 \equiv \beta$, the only possible solution is $\chi = \text{const}$, $\rho = \text{const} \equiv \rho_0$, so that $\beta = -\rho_0^2$. The next step is to consider the effect of small ϵ, still keeping $\kappa = 0$. Following the lines of the analysis developed in Ref. [3], it is easy to demonstrate that, in the lowest nontrivial approximation, one obtains an equation for the angular variable $\chi(\tau)$, simply freezing the radial variable in the Hamiltonian (5), i.e., setting there $\rho \equiv \rho_0$. Subsequent elementary consideration of the resultant equation for χ [3] yields the solution corresponding to the DW:

$$\chi(\tau) = \frac{1}{4}\pi - \tan^{-1}\left[\exp(-s\sqrt{\epsilon}\rho_0\tau)\right]\,, \quad (7)$$

where $s = \pm 1$ is the *polarity* of the DW. Obviously, equivalent DW solutions can be obtained from Eq. (7) by the shift

$$\chi(\tau) \to \chi(\tau) + \frac{1}{2}\pi n \quad (8)$$

with an arbitrary integer n. At the next order of the formal expansion in powers of $\sqrt{\epsilon}$, one can easily obtain a correction to ρ_0:

$$\rho_1(\tau) = (2\rho_0)^{-1}\left[\left(\frac{d\chi}{d\tau}\right)^2 - \frac{1}{2}\epsilon\rho_0^3\cos^2(2\chi)\right] \equiv -(2\rho_0)^{-1}\left(\frac{d\chi}{d\tau}\right)^2\,, \quad (9)$$

where the particular form of the solution (7) has been used.

The solution (7) connects the asymptotic values of the angular variable $\chi(\tau = -\infty) = -\frac{1}{2}\pi$ and $\chi(\tau = +\infty) = +\frac{1}{2}\pi$. Coming back to Eq. (6), one notices that $U^2(\tau = \pm\infty) = V^2(\tau = \pm\infty)$, and the fundamental property of the corresponding DW is changing the sign of one component V while keeping the sign of the other component U fixed. The DW solution generated by the transformation (8) with odd n changes the sign of U, while the sign of V is kept fixed. The homogeneous phases with $U = \pm V$ correspond to the linear combinations of the orthogonal linear polarizations, i.e., the *elliptic* polarizations [the ellipticity is determined by the arbitrary phase constant ϕ_0 in Eqs. (4)]. Thus the DW separates two elliptic polarizations with the relative phase difference $\Delta\chi = \frac{\pi}{2}$. As a matter of fact, this exactly corresponds to the DW solution recently found numerically in Ref. [7] directly in terms of the circular basic polarizations.

If the propagation constants in Eqs. (4) are slightly different, i.e., $\beta_{1,2} = \beta \pm \gamma$ with $|\gamma| \ll |\beta|$, the analysis based on the Hamiltonian (5) reveals that the small parameter γ introduces an asymmetry between the solutions corresponding to even and odd n in Eq. (8). At $n = 0$, the solution modified by a small $\gamma \neq 0$ connects the asymptotic values $\chi(\pm\infty) = \pm[\frac{\pi}{4} - \gamma(\epsilon\rho_0^2)^{-1}]$, so that the phase difference across the DW's with even values of n is now $\frac{\pi}{2} - 2\gamma\left(\epsilon\rho_0^2\right)^{-1}$. Similarly, it is easy to find that the phase difference across the DW's with odd n is $\frac{\pi}{2} + 2\gamma\left(\epsilon\rho_0^2\right)^{-1}$.

Really interesting results can be obtained when the twist is included. First of all, the analysis of the Hamiltonian (5) with $\kappa \neq 0$ shows that the stationary solutions considered above, with the phase difference $\Delta\chi = \frac{\pi}{2}$, no longer exist. Instead, a stationary solution with $\Delta\chi = \pi$ appears. In the case of small κ, the solution can be regarded as a bound state of two former solutions, with even and odd n. Actually, in this case the Hamiltonian (5) (with ρ frozen and with $\beta_1 = \beta_2$) coincides with that of the known double sine-Gordon (DSG) model (see the review paper [15]). In particular, the stationary solution in the form of the bound state corresponds to the well-known 4π-kink static solution of the DSG model. In the case when κ is small enough, the separation $\Delta\tau$ between two DW's in the bound state can be easily found using the well-known results for the DSG model [15]:

$$\Delta\tau \approx \left(\sqrt{\epsilon}\rho_0\right)^{-1}\ln\left(\kappa\rho_0^2\right)^{-1}\,. \quad (10)$$

In the presence of the twist, the usual DW does not exist as a stationary solution because the twist breaks the symmetry between the two different elliptic polarizations separated by the DW. One of the polarizations gets a larger Hamiltonian density than another; hence it cannot exist as a stable homogeneous phase. However, a sandwich in the form of a domain of the "bad" polarization squeezed between semi-infinite domains of the "good" polarization can exist and be stable, and this is exactly the bound state of the two DW's described above. Note that the temporal width of the sandwich, given by Eq. (10), is determined by the background power of the light beam (i.e., ρ_0^2) and by the twist parameter κ. Thus it should be easy to control the width in experiments with the fibers.

Coming back to the solitary DW, an interesting problem is to consider its actual dynamics in the presence of the twist. Although the DW cannot exist in the stationary state, one may expect that it may exist moving with a constant acceleration, like the soliton in the monomode fiber "accelerated" by the intrapulse Raman effect [8] or like the 2π kink in the DSG model [15]. However, a full solution of this problem cannot be borrowed from the DSG model and it should be found directly in the framework of Eqs. (1) and (2). The simplest approach is based on the so-called balance equation for the wave momentum of the system,

$$P = i\int_{-\infty}^{+\infty} (uu_\tau^* + vv_\tau^*)\,d\tau\,, \tag{11}$$

the asterisk standing for the complex conjugation. Differentiating the expression (11) in the evolutional variable z and directly using Eqs. (1) and (2), one obtains

$$\frac{dP}{dz} = -\kappa\,(u^*v + uv^*)\,|_{\tau=-\infty}^{\tau=+\infty}\,. \tag{12}$$

Equation (12) demonstrates that, if the expression on its right-hand side vanishes, the momentum is an integral of motion (in particular, this is true if the wave fields vanish at infinity). However, for the DW solution (7) the right-hand side of Eq. (12) does not vanish, and one eventually obtains

$$\frac{dP}{dz} = -2s\kappa\rho_0^2 \tag{13}$$

(remember that s is the polarity of the DW). As a matter of fact, Eq. (13) gives an effective driving force applied to the DW.

To make use of Eq. (13), it is necessary to know the momentum of the DW. However, inserting Eqs. (4) with U and V real into Eq. (7), one immediately obtains zero. To obtain a nontrivial result, the accelerated DW will be sought for in the form [cf. Eqs. (4)]

$$u(z,\tau) = a(\tau - \tau_0(z))\exp\{i\beta z + i\phi(\tau - \tau_0(z))\}\,, \tag{14}$$

$$v(z,\tau) = b(\tau - \tau_0(z))\exp\{i\beta z + i\psi(\tau - \tau_0(z))\}\,, \tag{15}$$

where the amplitudes a and b and the phases ϕ and ψ are real, $\tau_0(z)$ being the temporal coordinate of the DW regarded as a function of the evolutional variable z. Note that the "velocity" of the DW, $\frac{d\tau_0}{dz}$, as well as the dependence of the phases upon τ are produced by the small perturbation (twist). Therefore, $\frac{d\tau_0}{dz}$ and derivatives of the phases will be regarded as small quantities. Then, inserting Eq. (14) into Eq. (1) and collecting the small terms (they all contain an extra factor i in comparison with the usual nonsmall terms), one obtains an equation which, on multiplying it by a, can be conveniently represented in the form of a full derivative:

$$\frac{d}{d\tau}\left(-\frac{d\tau_0}{dz}a^2 + a^2\frac{d\phi}{d\tau}\right) = 0\,. \tag{16}$$

Integrating Eq. (16) yields

$$-\frac{d\tau_0}{dz}a^2 + a^2\frac{d\phi}{d\tau} = \text{const}\,. \tag{17}$$

The arbitrary constant of integration in Eq. (17) must be chosen so that to comply with vanishing of $\frac{d\phi}{d\tau}$ at infinity, where one has the homogeneous wave fields, const $= -\frac{d\tau_0}{dz}a_0^2$, where a_0 is the value of the amplitude at infinity, obviously related to the amplitude ρ_0 introduced above: $a_0^2 \equiv \frac{1}{2}\rho^2$. Eventually, one obtains

$$a^2\frac{d\phi}{d\tau} = -\frac{d\tau_0}{dz}\left(\frac{1}{2}\rho_0^2 - a^2\right)\,. \tag{18}$$

Quite similarly, one obtains from Eqs. (15) and (2) the relation

$$b^2\frac{d\psi}{d\tau} = -\frac{d\tau_0}{dz}\left(\frac{1}{2}\rho_0^2 - b^2\right)\,. \tag{19}$$

Finally, Eqs. (14), (15), (18), and (19) should be inserted into Eq. (11) to produce the following expression for the momentum of the DW:

$$P = -\frac{d\tau_0}{dz}\int_{-\infty}^{+\infty}\left(\rho_0^2 - a^2 - b^2\right)d\tau \equiv -\frac{d\tau_0}{dz}\int_{-\infty}^{+\infty}\left(\rho_0^2 - \rho^2\right)d\tau\,, \tag{20}$$

where the identity $a^2 + b^2 \equiv \rho^2$ was used [cf. Eqs. (4), (6), (14), and (15)].

As one sees from Eq. (20), the DW's momentum combines two small factors: the "velocity" $\frac{d\tau_0}{dz}$ and the small deviation of ρ from ρ_0 determined by Eq. (9). Finally, inserting Eq. (9) into Eq. (21), one obtains

$$P = -\frac{1}{2}\sqrt{\epsilon}\rho_0\frac{d\tau_0}{dz}\,. \tag{21}$$

It is noteworthy that, according to Eq. (21), the effective "mass" of the DW, defined as the proportionality coefficient between the momentum and the "velocity," is negative. For comparison, if one considers the momentum density of the homogeneous cw solution, it is straightforward to find out that the same definition (11) of the momentum yields a *positive* mass density of the homogeneous wave, so that the minus sign in Eq. (21) is not produced merely by an irrelevant definition. Actually, the negative mass is related to the fact that one is dealing not with a solitary wave but with a localized wave on the background of the cw solution.

The equation of motion sought for can now be obtained by substituting Eq. (21) into the left-hand side of Eq. (13), which eventually yields

$$\frac{d^2\tau_0}{dz^2} = 4s\epsilon^{-1/2}\rho_0\kappa\,. \tag{22}$$

Formally, Eq. (22) is the equation of motion for a classical particle driven by a constant force. It is similar to the well-known equation of motion for the Raman-driven soliton in the monomode fiber [8]. However, in that case the sign of the "acceleration" is fixed, as the intrapulse Raman scattering gives rise to the downshift of the soliton, but not to its upshift, while in the present case the signs are different for $s = \pm 1$. In applications, the DW's law of motion (22) can be controlled by means of the background amplitude ρ_0 or by changing the twist.

Since the direction of motion is different for the opposite polarities, collisions between DW's are possible. Although a full analysis of the collision may be complicated, it seems evident that the collision must result in the formation of the bound state of the two DW's described above.

As it was said above, the underlying equations (1) and (2) did not take into account the effective birefringence of the linear polarizations. This effect is accounted for by the additional terms $\Omega u + i\delta u_\tau$ and $-\Omega v - i\delta v_\tau$ in Eqs. (1) and (2), respectively, where Ω and δ are proportional to the birefringence-induced phase velocity and group velocity differences between the two linear polarizations [17]. Obviously, these terms can be excluded by the transformation $u \to u \exp\left[-i\delta\tau + i(\Omega + \frac{1}{2}\delta^2)z\right]$, $v \to v \exp\left[i\delta\tau + i(-\Omega + \frac{1}{2}\delta^2)z\right]$, which, however, adds the oscillating multipliers $\exp\left[\pm 2i(\delta\tau - \Omega z)\right]$ to the former constant κ. Looking at this additional multiplier, one infers that the influence of the birefringence is negligible if the corresponding temporal and spatial beating scales δ^{-1} and Ω^{-1} are large in comparison, respectively, with the characteristic temporal size of the DW (7) and with the length of the system; otherwise, the birefringence will strongly attenuate the effects considered above.

To conclude this section, it is relevant to note that the analysis can be easily extended to the case $\epsilon < 0$, or, according to Eq. (3), $\sigma > 1$ in Eqs. (1) and (2). Actually, the case of physical interest is $\sigma = 2$, i.e., $\epsilon = -1$, which corresponds to interaction of two circular polarizations or of two waves with different carrier wavelengths. This value of ϵ, unlike the value $\epsilon = \frac{1}{3}$ dealt with above, can scarcely be treated as a small parameter. Nevertheless, it may be relevant to discuss at least qualitative results which follow from the Hamiltonian (5) with $\epsilon < 0$ and with ρ frozen. In the system of two coupled circular polarizations, the terms in the effective Hamiltonian (5) proportional to the coefficients $\beta_1 - \beta_2$ and κ have physical meaning different from that in the system of the coupled linear polarizations. The former term accounts for the effective birefringence of the circular polarizations which is produced by the twist; the latter term accounts for the linear coupling between the two circular polarizations produced, in the absence of the twist, by the birefringence of the corresponding linear polarizations [11]. Then, it is easy to see that, at $\epsilon < 0$, the term proportional to $\beta_1 - \beta_2$ in the Hamiltonian (5) gives rise to an effective constant force moving the DW, while the term proportional to κ introduces a discrimination between the DW's corresponding to even and odd n in Eq. (8), but does not make them to move. With regard to the abovementioned meaning of the parameters $\beta_1 - \beta_2$ and κ at $\epsilon < 0$, it is evident that these qualitative results comply with those obtained above in the quantitative form for the linear polarizations, as well as with the numerical results for the system of the coupled circular polarizations reported in Ref. [7].

III. THE RAMAN-DRIVEN OPTICAL DOMAIN WALL

In this section, Eqs. (1) and (2) without the twist-induced linear coupling but with additional terms accounting for the stimulated Raman scattering in the bimodal fiber will be considered. In the simplest approximation based on the assumption of the quasi-instantaneous response of the medium [9], the corresponding equations take the form [9]

$$iu_z + i\delta u_\tau + \frac{1}{2}u_{\tau\tau} - \left(|u|^2 + \sigma|v|^2\right) = \lambda_1\left(|u|^2\right)_\tau u + \lambda_2\left(|v|^2\right)_\tau u + \lambda_3\left(uv^*\right)_\tau v\,, \quad (23)$$

$$iv_z - i\delta v_\tau + \frac{1}{2}v_{\tau\tau} - \left(|v|^2 + \sigma|u|^2\right) = \lambda_1\left(|v|^2\right)_\tau v + \lambda_2\left(|u|^2\right)_\tau v + \lambda_3\left(vu^*\right)_\tau u\,, \quad (24)$$

where δ is the same birefringence coefficient as in the preceding section. Note that, since the twist is not considered in this section, the other birefringence coefficient Ω can always be excluded from Eqs. (23) and (24). The coupling constant $\sigma \equiv 1 - \epsilon$ is again $\frac{2}{3}$. Finally, the *parallel* and *perpendicular* Raman coefficients λ_1 and $\lambda_{2,3}$ in Eqs. (23) and (24) are always related by the fundamental equation following from the isotropy of nonlinearity and reality of polarizability in the optical medium [9]:

$$\lambda_1 = \lambda_2 + \lambda_3\,. \quad (25)$$

An effective equation of motion for the DW (7), with the Raman terms (and the birefringence term) regarded as small perturbations, can be again derived from the momentum balance. First of all, it is convenient to exclude the birefringence term from Eqs. (23) and (24) by means of the transformation

$$\begin{aligned} u(\tau,z) &\equiv U(\tau,z)\exp\left(-i\delta\tau + \frac{1}{2}i\delta^2 z\right)\,,\\ v(\tau,z) &\equiv V(\tau,z)\exp\left(+i\delta\tau + \frac{1}{2}i\delta^2 z\right)\,, \end{aligned} \quad (26)$$

which leads to equations for U and V similar to Eqs. (23) and (24), but without the birefingence terms and with the additional terms, respectively, $-2i\delta\lambda_3|V|^2U$ and $+2i\delta\lambda_3|U|^2V$ on the right-hand sides. Finally, differentiating the momentum (11) in z and taking into account all the Raman terms, one can obtain a general expression for $\frac{dP}{dz}$; cf. Eq. (12). To further simplify this expression, note that, in the lowest approximation, one can take purely real U and V, as it was done in the preceding section when deriving Eq. (13). Thus one obtains

$$\frac{dP}{dz} = \int_{-\infty}^{+\infty}\{(\lambda_2 + 2\lambda_3)\left[(U^2)_\tau^2 + (V^2)_\tau^2\right] + 2\lambda_2(U^2)_\tau(V^2)_\tau + 2\lambda_3(UV)_\tau^2\}d\tau\,. \quad (27)$$

The relation (25) was used to exclude λ_1 from Eq. (21). Next, inserting the representation (6) into Eq. (27), and then using the DW solution (7) with ρ frozen ($\rho \equiv \rho_0$), it is easy to obtain the eventual expression for the effective force driving the DW:

$$\frac{dP}{dz} = \frac{4}{3}\sqrt{\epsilon}\lambda_3\rho_0^5\,. \quad (28)$$

Notice that only the coefficient λ_3, which is the single one accounting for the Raman-induced energy transfer between the two polarizations [9], shows up in Eq. (28).

In the approximation considered, one can take for the DW's momentum, regarded as a function of the velocity $\frac{d\tau_0}{dz}$, the same expression (21) which was used in the preceding section. The eventual equation of motion for the DW, which follows from Eqs. (28) and (21), is

$$\frac{d^2\tau_0}{dz^2} = -\frac{8}{3}\lambda_3\rho_0^4\,. \quad (29)$$

This equation is quite similar to that describing the motion of the soliton in the single-mode fiber in the presence of the Raman effect [8]. Note that, unlike Eq. (22), the sign of the right-hand side of Eq. (29) does not depend on the DW's polarity, as this sign is actually fixed by the physical condition that the Raman scattering transfers energy to longer wavelengths [8]. An important consequence of this is the fact that the DW's driven by the Raman force do not collide, and, accordingly, their "sandwichlike" bound state does not exist. It is also worthy to note that the Raman-induced force driving the DW can be easily compensated by the twist of the proper sign.

IV. DARK SOLITONS IN THE DIRECTIONAL COUPLER

The simplest model of the directional coupler, i.e., a dual-core optical fiber, is based on the following coupled NLS equations [10]:

$$iu_z + \frac{1}{2}u_{\tau\tau} - |u|^2 u + \kappa v = 0\,, \tag{30}$$

$$iv_z + \frac{1}{2}v_{\tau\tau} - |v|^2 v + \kappa u = 0\,, \tag{31}$$

where this time κ is the coefficient of the tunnel coupling between the two cores. As well as in the preceding sections, the choice of the signs in Eqs. (30) and (31) corresponds to the fibers with the normal dispersion, so that stable dark solitons (DS's) (see the review [18]) can exist in each core in absence of the coupling. The corresponding solution is [18]

$$u = \rho_0 \exp\left(-i\rho_0^2 z\right) \tanh(s\rho_0 \tau)\,, \tag{32}$$

where $s = \pm 1$ is the polarity of the DS and ρ_0 is the background field amplitude.

In the presence of the coupling, the DS or a solution close in form to it cannot exist. Actually, the field configuration consisting of the DS in one core and the homogeneous field in the second core is another example of the optical DW driven by an effective constant force. Indeed, differentiating the momentum (11) in z and substituting Eqs. (30) and (31), one obtains for $\frac{dP}{dz}$ exactly the same expression (12) which was already obtained above. Next, it immediately follows from Eqs. (30) and (31) that, at $\tau \to \pm\infty$ (i.e., far from the location of the DS), the wave fields in the two cores must be related as follows: $u = \pm v$. For the definiteness, in what follows the upper sign in this relation will be taken at $\tau = +\infty$. Then, the effective force driving the DS in the first core due to its interaction with the homogeneous (τ-independent) field in the second core can be obtained from Eq. (12) in the form

$$\frac{dP}{dz} = -4s\rho_0^2\kappa\,; \tag{33}$$

cf. Eq. (13). To find the momentum of the "slowly moving" DS, one can again use Eq. (20), with $\rho(\tau)$ substituted, according to Eq. (32), by $\rho_0 \tanh(s\rho_0\tau)$. An elementary calculation yields [cf. Eq. (21)]

$$P = -2\rho_0 \frac{d\tau_0}{dz}\,. \tag{34}$$

Note that the effective mass corresponding to Eq. (34) is negative, as well as that in Eq. (21). Finally, the DW's law of motion following from Eqs. (34) and (35) is

$$\frac{d^2\tau_0}{dz^2} = 2\sigma\rho_0\kappa\,; \tag{35}$$

cf. Eqs. (22) and (29). As it follows from Eq. (35), one can easily control the effect varying the background amplitude ρ_0.

Since the DS's with the opposite polarities move in the opposite directions according to Eq. (35), they may collide. The collision will lead to formation of a bound state of the DS's. Indeed, it is well known that two DS's in the single-core fiber repel each other [19], the repulsion force being $\sim \exp(-2\rho_0\Delta\tau)$, where $\Delta\tau$ is the temporal separation between the DS's. Combining this force and that given by Eq. (35), it is easy to find, with the logarithmic accuracy [i.e., when $\ln(\rho_0\Delta\tau)$ is a large quantity], that the pair of the DS's belonging to the same core of the coupler form a stable bound state with the separation between them

$$\Delta\tau \approx \frac{1}{2}\rho_0^{-1}\ln(\kappa\rho_0^2)^{-1}\,; \tag{36}$$

cf. Eq. (10). Following Sec. II, one can can consider this bound state as a sandwich, with the sign of the u field in the inner domain opposite to that in the outer domains. Contrary to this, the difference of values of the v field in the inner and outer domains is a small quantity $\sim \kappa$.

V. CONCLUSION

In this work, domain walls in the light beams propagating in nonlinear optical fibers were considered. Two particular types of the DW were dealt with: the one between different circular (or, strictly speaking, elliptic) polarizations in the bimodal fiber and a dark soliton in one core of the dual-core coupler. A salient feature of both types of the DW's is the existence of a fundamental "force" which makes them to move: the fiber's twist in the former case or the interaction with the mate core in the latter case. The sign of this effective force depends upon the DW's polarity. In absence of the twist, the Raman effect can also give rise to an effective force driving the DW in the bimodal fiber, which, however, does not depend upon the polarity. In all the cases, an effective equation of motion of the driven DW was derived by means of the momentum-balance technique which was a nontrivial problem in itself, its noteworthy peculiarity being the negative effective mass of the DW. Pairs of the DW's with the opposite polarities, driven by the twist in the bimodal fiber or by the interaction with the mate core in the coupler, may collide and form stable bound states. These bound states are "sandwiches," with a domain of a certain phase squeezed between semi-infinite domains of another phase. In principle, the sandwiches may find a practical application, e.g., for the optical storage and transfer of information. The temporal width of the sandwiches was calculated in this work in the logarithmic approximation. It can be easily controlled using the background amplitude of the light beam or, e.g., the fiber's twist.

Finally, it is relevant to mention that, although the consideration in this work was accomplished in terms of the nonlinear optical fibers, some of the results can be reformulated in terms of the nonlinear planar lightguides (see, e.g., Ref. [7]), i.e., in the spatial domain instead of the temporal one. However, the nonlinear fibers seem more promising for possible experiments.

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